**Barron’s Let’s Review Regents – Algebra II**

# Chapter 3: Exponential and Logarithmic Expressions and Equations

## 3.1 Properties of Exponents

**Key Ideas**

An expression like is called an *exponential expression*. It has a *base*, which in this case is the number 2, and an *exponent*, which in this case is the number 5. For positive integer exponents, the expression can be evaluated by multiplying the base by itself the number of times of the exponent.

**Multiplying Exponential Expressions**

When two exponential expressions that have the same base are multiplied, the product can be written as an exponential expression that has that base but whose exponent is the sum (not the product) of the exponents. If the bases are different, there is no simple way to multiply the expressions.

**Example 1**

Simplify .  
(3)

**Dividing Exponential Expressions**

When two exponential expressions that have the same base are divided, the product can be written as an exponential expression that also has that base but whose exponent is the difference of the exponents. If the bases are different, there is no simple way to divide the expressions.

**Example 2**

Simplify .  
(4)

**Raising a Power to a Power**

To raise an exponential expression to a power, keep the same base as in the original expression. Multiply the two exponents to find the new exponent.

**Example 3**

Simplify .

**Math Facts**

There are three main properties of exponents.

**Math Facts**

Any number (besides 0) raised to the 0 power is equal to 1. Any number (besides 0) raised to the negative power is equal to the reciprocal of that number raised to the positive version of that power.

**Math Facts**

To raise a base to a fractional power , take the *d*th root of the number and raise it to the *n*th power.

**Example 7**

What is the value of ?

**The Distributive Property for Exponents**

If an expression in parentheses is the product of numbers and/or variables, the entire expression in the parentheses can be raised to a power by raising each of the factors to that power and multiplying them together.

For example, cam be calculated by   
. This is also the solution if you simplified inside the parentheses first:   
. This property is needed when variables are involved.

**Example 9**

Simplify .

### Check Your Understanding of Section 3.1

1. Multiple-Choice
2. Simplify .  
   **(2)**
3. Simplify .  
   **(2)**
4. What is ?  
   **(2)**
5. What is ?  
   **(4)**
6. Simplify .  
   **(3)**
7. What is ?  
   **(1)**
8. What is ?

**(2) 1**

1. What is ?  
   **(3)**
2. What is   
   **(3) 49**
3. What is ?  
   **(2) 1**
4. *Show how you arrived at your answers*.
5. Ashlynn says that . Colin says that it is equal to Who is right and why?  
     
   Ashlynn is right because:
6. In 5th grade, Charles learned that   
   . Show how the properties of negative exponents justifies this answer.
7. If , how can you quickly calculate if your calculator does not have an exponent key?
8. If and , what is the value of ? Hint: change 1.5 into an improper fraction.)
9. What is the value of ?

## 3.2 Solving Exponential Equations By Guess and Check Or By Graphing

**Key Ideas**

An *exponential equation* is one in which the variable is an exponent. An example is the equation . Some exponential equations have integer solutions, some have rational solutions (fractions), and some have irrational solutions. One way to solve exponential equations is through guess and check. Another way is to use the intersect feature of a graphing calculator.

**Solving Exponential Equations with Guess and Check**

**Example 1**

Solve by guess and check.

*Solution*: Since , the solution is 6.

**Example 2**

Which value of makes

*Solution*: Since is between 0 and 1, the answer must be negative.

Since , the denominator is too small. Since , the answer is .

**Example 3**

To the nearest tenth, what is the solution to ?

*Solution*: Since , which is too small, and since , which is too big, the answer is between 5 and 6. Test the numbers 5.1, 5.2, 5.3 and so on until you find a number that is close to 42.

|  |  |
| --- | --- |
| x |  |
| 5.1 | 34.3 |
| 5.2 | 36.8 |
| 5.3 | 39.4 |
| 5.4 | 42.2 |

Sometimes it is possible to get an exact solution even when the solution is not an integer. In those cases, the exponent is a rational number (a fraction).

**Example 4**

What is the exact solution to ?

*Solution*: Since and , the answer is between 2 and 3.

Since , and , .

Exponential equations with exponents on both sides of the equal sign are easiest to solve when the two bases are the same.

**Example 5**

What value of makes ?

*Solution*: Since the bases are the same, the exponents must be equal. So .

Example 6

What value of makes ?

*Solution*: Since the bases are the same, the exponents must be equal. This means that   
. This can be solved with algebra: .

If the bases are different, they can sometimes be converted into the same base and then solved by equating the exponents.

**Example 7**

What value of makes ?

*Solution*: Since 9 can be written as , the right side of the equation becomes . This is equivalent to because powers can be raised to powers by multiplying the exponent.

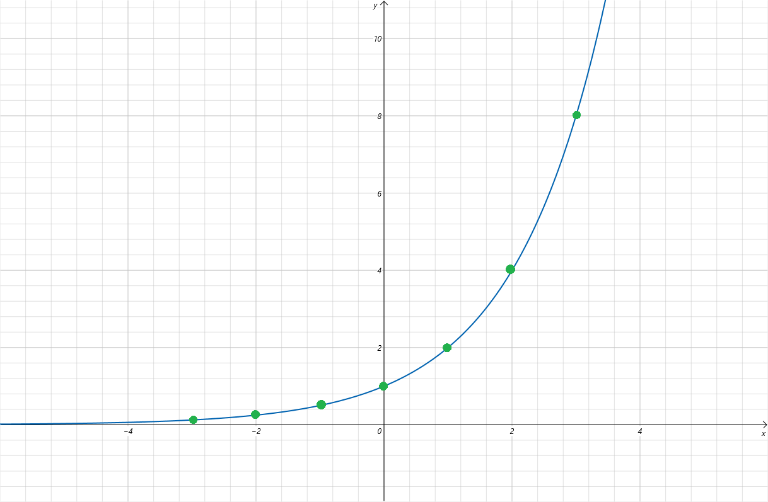
This equation is now .

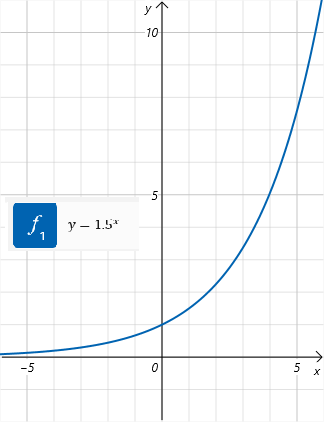
**Graphs of Two-Variable Exponential Equations**

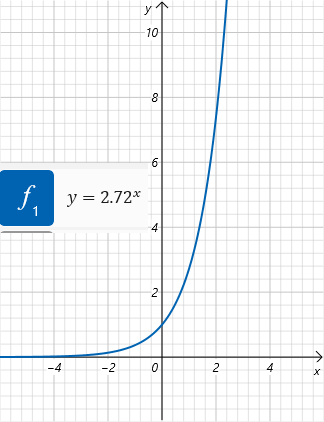
An equation of the form , where a, b, and are numbers is a *two-variable exponential equation.* Like all two-variable equations, the solution set is a set of ordered pairs that make the equation true.

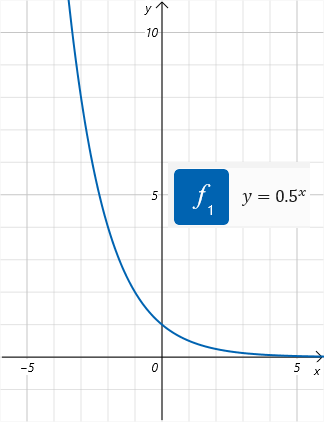
For example, the equation has ordered pairs (0, 1), (1, 2), (2, 4), and (3, 8) as four elements of the solution. Other points can be found by creating a chart where different integer values are chosen for and the -value is then calculated.

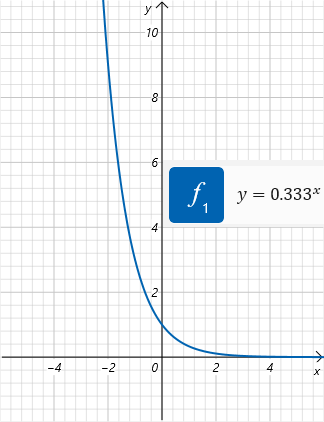
|  |  |
| --- | --- |
| x |  |
| 3 | 8 |
| 2 | 4 |
| 1 | 2 |
| 0 | 1 |
| -1 |  |
| -2 |  |
| -3 |  |





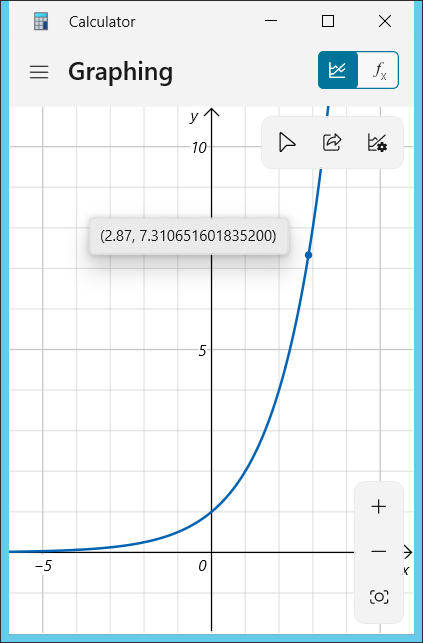






**Solving Exponential Equations with a Graphing Calculator**

When an exponential equation has an answer that is not an integer, one way to get a approximate answer is to use the intersect feature of the graphing calculator.

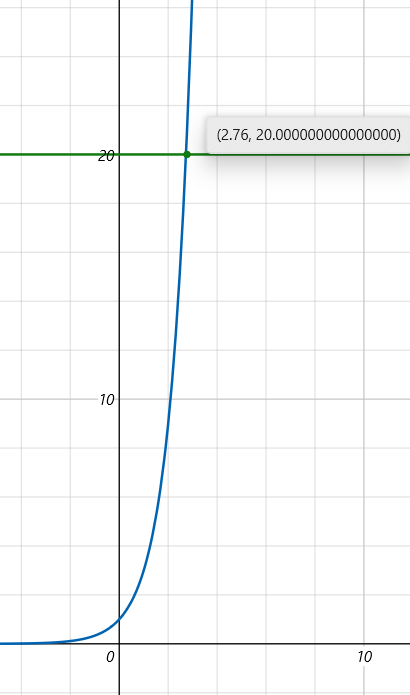


### Check Your Understanding of Section 3.2

1. Multiple-Choice
2. Solve .  
   **(1) 4**
3. Solve .  
   **(2) -2**
4. Which value is nearest to if .  
   **(4) 3.4**

|  |  |
| --- | --- |
|  |  |
| 3 | 27 |
| 3.2 | 33.6 |
| 3.4 | 41.9 |
| 4 | 81 |

1. Solve .  
   ,   
      
   **(1)**
2. Solve .  
   **(3) 0**
3. Solve .  
   **(2) -2**
4. Solve **(4) 4**
5. Solve .  
   Both must have an exponent of 0, so the result is 1.  
   **(4) -4**
6. Which of the following is a graph of ?  
   (1) or (4) because negative exponents have values less than 1, and positive exponents have values greater than 1. And .  
   , Point: (2, 9)  
   **(4)**
7. The graph of which equation is shown?  
   **(3)**
8. *Show how you arrived at your answers*.
9. Below is a graph of and . Based on the graph, what is an approximate solution to the equation .  
   A screenshot of a math problem

   AI-generated content may be incorrect.  
   **Estimated from book graph:   
   Corrected:   
   Correct answer: 2.73**
10. Charles says this is the graph . Hope says that it is the graph of ? Who is correct?  
    **Both are correct, because .**
11. Solve for if .
12. is between which two integers. Explain your reasoning.

|  |  |
| --- | --- |
| x |  |
| 3 | 64 |
| 4 | 256 |

is too small (64) and is too big.

1. Bianca is solving the equation as follow:  
     
   **It is not correct to multiply an exponential expression by a term that does not share the same base. She should have divided both sides by 3, removing the 3 term from the left hand side.  
     
   “When two exponential expressions that have the same base are multiplied, the product can be written as an exponential expression that has that base but whose exponent is the sum (not the product) of the exponents. If the bases are different, there is no simple way to multiply the expressions”**  
   Divide both sides by 3:Solving graphically:  
   , , solution: x = 3.58  
   